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Study of Free Vibrations in Isotropic Poroelastic Solid Sphere with Rigidly Fixed Conditions

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Abstract

This paper deals with free vibrations in isotropic poroelastic solid sphere with rigidly fixed boundary conditions. The frequency equation for pervious surface is obtained in the framework of Biot's theory of wave propagation. For illustration purpose, three materials, namely sandstone saturated with kerosene, sandstone saturated with water, and a bone material have been used. Results are presented graphically.

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Keywords: Spherical shell; Phase velocity; wave number; Frequency equation; Flexural vibration.

1. Introduction

Spherical solids are fundamental and core structural elements in various fields of engineering technology. Kumar [1] investigated the axially symmetric vibrations of fluid filled spherical shells. The radial vibrations in poroelastic sphere are investigated by Paul [2]. In this paper, Paul obtained frequency equations for the free radial vibrations of a sphere and a spherical shell. Torsional vibrations of poroelastic spheroid shells are studied by Shah and Tajuddin [3] on the framework of Biot's theory [4]. In said paper, authors derived frequency equations for poroelastic thin spherical shell, thick spherical shell, poroelastic solid sphere. Flexural vibrations of poroelastic elliptic cone are investigated by Rajitha and Malla Reddy [5]. In said paper, authors derived frequency equations for poroelastic

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elliptic cone against the angle made by the major axis of the cone in the spheroconal co-ordinate system. Vibrations analysis of a poroelastic composite hollow sphere is investigated by Shanker. et al. [6]. In this paper, authors discussed radial and rotatory vibrations of fluid filled and empty poroelastic shells with rigid core. Employing Biot's theory of poroelasticity, ring modes in poroelastic spherical shell are discussed in the paper Gazis., [7]. Flexural vibrations in poroelastic solid sphere with rigidly fixed condition shell are not yet investigated. Therefore, in this paper, the same is investigated in frame work of Biot's theory.

The rest of the paper is organized as follows. In section 2, governing equations and solutions of the problem are given. Boundary conditions and frequency equations are derived in section 3, numerical results are described in section 4, finally, conclusion is given in section 5.

2. Governing equation and solution of problem

The equations of motion of a homogeneous, isotropic poroelastic solid Biot's theory [4] in the presence of dissipation (b) are

$$N\nabla^2\vec{u} + (A+N)\nabla e + Q\nabla\varepsilon = \frac{\partial^2}{\partial t^2}\left(\rho_{11}\vec{u} + \rho_{12}\vec{U}\right) + b\frac{\partial}{\partial t}\left(\vec{u} - \vec{U}\right),$$

$$Q\nabla e + R\nabla\varepsilon = \frac{\partial^2}{\partial t^2}\left(\rho_{12}\vec{u} + \rho_{22}\vec{U}\right) - b\frac{\partial}{\partial t}\left(\vec{u} - \vec{U}\right). \tag{1}$$

where ∇^2 is the Laplacian operator, $u(u,v,w)$ and $U(U,V,W)$ are displacements of the solid and fluid media, respectively, while e and ε are dilatations of solid and fluid, respectively. A,N,Q,R are all poroelastic constants and $\rho_{11},\rho_{12},\rho_{22}$ are the mass coefficients [4].

The constitutive relations here are

$$\sigma_{kl} = 2Ne_{kl} + (Ae + Q\varepsilon)\delta_{kl}, \quad k,l = r,\theta,\phi.$$

$$s = Qe + R\varepsilon. \tag{2}$$

Here e_{kl} are strain components of the poroelastic solid [4] and δ_{kl} is the well-known Kronecker delta function. Consider an isotropic poroelastic sphere with radius a in spherical coordinate system (r,θ,ϕ). Let $\vec{u}(u_r,u_\theta,u_\phi)$ and $\vec{U}(U_r,U_\theta,U_\phi)$ be the displacements vectors of solid and fluid respectively. For flexural vibrations, the displacements potentials of $\phi's$, $h's$ and $H's$ which are functions of r,θ,ϕ and t are introduced as follows [5].

$$u_r = \frac{\partial\phi_1}{\partial r} + \frac{\sqrt{M_1}}{r^2T_1}\left(\frac{\partial}{\partial\theta}\left(rh_\phi\sqrt{\frac{T_1}{M_1}}\right) - \left(\frac{\partial}{\partial\phi}\left(rh_\theta\sqrt{T_1}\right)\right)\right),$$

$$u_\theta = \frac{1}{r\sqrt{T_1}}\left(\sqrt{M_1}\frac{\partial\phi_1}{\partial\theta} + \left(\frac{\partial h_r}{\partial\phi} - \frac{\partial}{\partial r}\left(r\sqrt{T_1}h_\phi\right)\right)\right),$$

$$u_\phi = \frac{1}{r\sqrt{T_1}}\left(\frac{\partial\phi_1}{\partial\phi} - \sqrt{M_1}\left(\frac{\partial h_r}{\partial\theta} - \frac{\partial}{\partial r}\left(r\sqrt{T_1}h_\theta\right)\right)\right),$$

$$U_r = \frac{\partial\phi_2}{\partial r} + \frac{\sqrt{M_1}}{r^2T_1}\left(\frac{\partial}{\partial\theta}\left(rH_\phi\sqrt{\frac{T_1}{M_1}}\right) - \left(\frac{\partial}{\partial\phi}\left(rH_\theta\sqrt{T_1}\right)\right)\right),$$

$$U_\theta = \frac{1}{r\sqrt{T_1}}\left(\sqrt{M_1}\frac{\partial \phi_2}{\partial \theta} + \left(\frac{\partial H_r}{\partial \phi} - \frac{\partial}{\partial r}\left(r\sqrt{T_1}H_\phi\right)\right)\right),$$

$$U_\phi = \frac{1}{r\sqrt{T_1}}\left(\frac{\partial \phi_2}{\partial \phi} - \sqrt{M_1}\left(\frac{\partial H_r}{\partial \theta} - \frac{\partial}{\partial r}\left(r\sqrt{T_1}H_\theta\right)\right)\right).$$

$$T_1 = \sin^2(\phi) + \left(\sin^2(\theta)\cos^2(\phi) - \sin^2(\phi)\cos^2(\theta)\right),$$

$$M_1 = 1 - \cos^2(\theta). \tag{3}$$

For free harmonic vibrations, the potential functions $\phi^{'}s$ and $\psi^{'}s$ are expressed as follows:

$$\phi_1 = f_1(r)p_n^m(\cos(\theta))\cos m\phi e^{i\omega t}, \qquad \phi_2 = f_2(r)p_n^m(\cos(\theta))\cos m\phi e^{i\omega t},$$

$$\vec{\psi}_1 = (h_r, h_\theta, h_\phi), \qquad \vec{\psi}_2 = (H_r, H_\theta, H_\phi),$$

$$h_r = g_r(r)p_n^m(\cos(\theta))\sin m\phi e^{i\omega t}, \qquad H_r = G_r(r)p_n^m(\cos(\theta))\sin m\phi e^{i\omega t}.$$

$$h_\theta = g_\theta(r)p_n^m(\cos(\theta))\cos m\phi e^{i\omega t}, \qquad H_\theta = G_\theta(r)p_n^m(\cos(\theta))\cos m\phi e^{i\omega t},$$

$$h_\phi = g_3(r)p_n^m(\cos(\theta))\sin m\phi e^{i\omega t}, \qquad H_\phi = G_3(r)p_n^m(\cos(\theta))\sin m\phi e^{i\omega t}, \tag{4}$$

here ω is the frequency of wave. The expression $p_n^m(\cos(\theta))$ is the associated Legendre polynomial, where n is the order of spherical harmonic $m = 0,1,\ldots n,$ i is the complex unity, and t is time. Substitution Eq. (3) into the equations of motion gives the equations of motions of displacements potential functions as follows [5].

$$P\nabla^2\phi_1 + Q\nabla^2\phi_2 = \left(\rho_{11}\ddot{\phi}_1 + \rho_{12}\ddot{\phi}_2\right) + b\left(\dot{\phi}_1 - \dot{\phi}_2\right),$$

$$Q\nabla^2\phi_1 + R\nabla^2\phi_2 = \left(\rho_{12}\ddot{\phi}_1 + \rho_{22}\ddot{\phi}_2\right) - b\left(\dot{\phi}_1 - \dot{\phi}_2\right),$$

$$N\nabla^2 h_\theta = \left(\rho_{11}\ddot{h}_\theta + \rho_{12}\ddot{H}_\theta\right) + b\left(\dot{h}_\theta - \dot{H}_\theta\right),$$

$$N\nabla^2 h_\phi = \left(\rho_{11}\ddot{h}_\phi + \rho_{12}\ddot{H}_\phi\right) + b\left(\dot{h}_\phi - \dot{H}_\phi\right),$$

$$N\nabla^2 h_r = \left(\rho_{11}\ddot{h}_r + \rho_{12}\ddot{H}_r\right) + b\left(\dot{h}_r - \dot{H}_r\right),$$

$$0 = \left(\rho_{12}\ddot{h}_r + \rho_{22}\ddot{H}_r\right) - b\left(\dot{h}_r - \dot{H}_r\right).$$

$$0 = \left(\rho_{12}\ddot{h}_\theta + \rho_{22}\ddot{H}_\theta\right) - b\left(\dot{h}_\theta - \dot{H}_\theta\right),$$

$$0 = \left(\rho_{12}\ddot{h}_\phi + \rho_{22}\ddot{H}_\phi\right) - b\left(\dot{h}_\phi - \dot{H}_\phi\right), \tag{5}$$

Substituting $\phi_1, \phi_2, h^{'}s$ and $H^{'}s$ in the Eqs. (5), then we obtain

$$P\Delta f_1 + Q\Delta f_2 = -\omega^2\left(M_{11}f_1 + M_{12}f_2\right),$$

$$Q\Delta f_1 + R\Delta f_2 = -\omega^2\left(M_{12}f_1 + M_{22}f_2\right),$$

$$N\Delta g_\theta = -\omega^2\left(M_{11}g_\theta + M_{12}G_\theta\right),$$

$$N\Delta g_3 = -\omega^2\left(M_{11}g_3 + M_{12}G_3\right),$$

$$N\Delta g_r = -\omega^2\left(M_{11}g_r + M_{12}G_r\right),$$

$$0 = -\omega^2\left(M_{12}g_\theta + M_{22}G_\theta\right),$$

$$0 = -\omega^2\left(M_{12}g_3 + M_{22}G_3\right),$$

$$0 = -\omega^2 \left(M_{12} g_r + M_{22} G_r \right). \tag{6}$$

Where $p = A + 2N$, $M_{11} = \rho_{11} - ib\omega^{-1}$, $M_{12} = \rho_{12} + ib\omega^{-1}$, $M_{22} = \rho_{22} - ib\omega^{-1}$,

$$\Delta = \frac{d^2}{dr} + \frac{2}{r}\frac{d}{dr} + \frac{1}{r^2}\left(\left(\frac{1}{T_1}\left(M_1 \sin^2(\theta) p_n^{m^{'}} \cos(\theta) - \cos(\theta)\sin^{2\theta} + M_1 \cos(\theta) \right) p_n^{m^{'}} \cos(\theta) \right) \frac{1}{p_n^m \cos(\theta)} \right) + m^2.$$

The solution of equations (6) is as follows:

$$f_1(r) = c_1 \frac{1}{\sqrt{pr}} J_l(pr) + c_2 \frac{1}{\sqrt{qr}} J_l(qr),$$

$$f_2(r) = c_1 \delta_1^2 \frac{1}{\sqrt{pr}} J_l(pr) + c_2 \delta_2^2 \frac{1}{\sqrt{qr}} J_l(qr),$$

$$g_3(r) = c_3 \frac{1}{\sqrt{dr}} J_l(dr),$$

$$2g_1(r) = g_r - g_\theta = 2c_4 J_l(dr),$$

$$2g_2(r) = g_r + g_\theta = 2c_5 J_l(dr). \tag{7}$$

In Eq. (7) c_1, c_2, c_3, c_4 and c_5 are constants. $J_1(x)$ is the spherical Bessel function of order l.

$$l = \frac{\left(L_{11} + L_{22}\right)^{\frac{1}{2}}}{2T_1}, \qquad L_{11} = T_1^2 + m^2,$$

$$L_{22} = 4T_1 \left(\left(\cos(\theta)\sin^2(\theta) + M_1 \cos(\theta) p_n^{m^{'}} \left(\cos(\theta)\right) - M_1 \sin^2(\theta) p_n^{m^{'}} \left(\cos(\theta)\right) \right) \right) \frac{1}{p_n^m \cos(\theta)},$$

$$\delta_1^2 = \frac{-\left(PR - Q^2\right)V_1^{-2}}{RM_{12} - QM_{22}} \left(\frac{1}{r^2\sqrt{pr}} \left(l^2 - p^2 r^2 - \frac{1}{4} + \frac{1}{T_1}\left(M_1 \sin^2(\theta) p_n^{m^{'}} \cos(\theta) - \left(\cos(\theta)\sin^2(\theta) + M_1 \cos(\theta) \right) \right) p_n^{m^{'}} \cos(\theta) \right) \right)$$
$$\frac{1}{p_n^{m^{'}} \cos(\theta)} - m^2 - \frac{RM_{11} - QM_{12}}{RM_{12} - QM_{22}} \frac{1}{\sqrt{pr}}. \tag{8}$$

$\delta_2^2 =$ Similar expression as δ_1^2 with p, V_1^{-2} replaced by q, V_2^{-2} respectively, and $p = \zeta_1$, $q = \zeta_2$, $d = \zeta_3$, $\zeta_i = \frac{\omega}{v_1}\ (i = 1, 2, 3.)$

The gauge invariance property [8] is used to eliminate one arbitrary constant c_5 (say) from the Eq. (4). Accordingly, any one of the potential functions g_1, g_2, g_3 can be set equal to zero. If we set $g_2 = 0$, then we obtain $g_r = -g_\theta = g_1$. Finally we remain with four arbitrary constants c_1, c_2, c_3 and c_4. Solid displacement components take the following form:

$$u_r = \left(C_1 M_{11}(r) + C_2 M_{12}(r) + C_3 M_{13}(r) + C_4 M_{14}(r) \right) \cos(m\phi) e^{i\omega t},$$
$$u_\theta = \left(C_1 M_{21}(r) + C_2 M_{22}(r) + C_3 M_{23}(r) + C_4 M_{24}(r) \right) \cos(m\phi) e^{i\omega t},$$
$$u_\phi = \left(C_1 M_{31}(r) + C_2 M_{32}(r) + C_3 M_{33}(r) + C_4 M_{34}(r) \right) \sin(m\phi) e^{i\omega t}, \tag{9}$$

where,

$$M_{11}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}\left[\left(\sin(\theta)\left(\frac{l}{r\sqrt{pr}}-\frac{1}{2\sqrt{pr^{\frac{3}{2}}}}\right)\right)J_l(pr)-\sqrt{\frac{p}{r}}J_{l+1}(pr)\right]P_n^m\cos(\theta),$$

$$M_{12}(r)=\left(T_1r\right)^{-\frac{1}{2}}\left[\left(\sin(\theta)\left(\frac{l}{r\sqrt{qr}}-\frac{1}{2\sqrt{qr^{\frac{3}{2}}}}\right)\right)J_l(qr)-\sqrt{\frac{q}{r}}J_{l+1}(pr)\right]q_n^m\cos(\theta),$$

$$M_{13}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}(T_1)^{-\frac{1}{2}}\sin(\theta)\frac{1}{2}\sin(2\theta)P_n^m\cos(\theta)-\frac{1}{2}\sin(2\theta)P_n^m\cos(\theta)-(T_1)^{\frac{1}{2}}P_n^{m'}\cos(\theta)\sin(\theta)+$$

$$\sin(m\phi)\frac{r}{\sqrt{dr}}J_l(dr),$$

$$M_{14}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}(T_1)^{-\frac{1}{2}}\frac{1}{2}\sin(2\phi)\tan(m\phi)+m(T_1)^{-\frac{1}{2}}\frac{r}{\sqrt{dr}}J_l(dr)P_n^m\cos(\theta),$$

$$M_{21}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}\left(-\sin(\theta)\frac{1}{\sqrt{pr}}J_l(pr)P_n^{m'}\cos(\theta)\sin(\theta)\right),$$

$$M_{22}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}\left(-\sin(\theta)\frac{1}{\sqrt{qr}}J_l(qr)q_n^{m'}\cos(\theta)\sin(\theta)\right),$$

$$M_{23}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}\left(m-(T_1)^{\frac{1}{2}}\tan(m\phi)\left(l+\frac{1}{2}\right)\frac{1}{\sqrt{dr}}J_l(dr)-\sqrt{dr}(T_1)^{\frac{1}{2}}\tan(m\phi)J_{l+1}(dr)P_n^m\cos(\theta)\right),$$

$$M_{24}(r)=0,$$

$$M_{31}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}\left(-m\frac{1}{\sqrt{pr}}J_l(pr)P_n^m\cos(\theta)\right),$$

$$M_{32}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}\left(-m\frac{1}{\sqrt{qr}}J_l(qr)q_n^m\cos(\theta)\right),$$

$$M_{33}(r)=\left(T_1r^2\right)^{-\frac{1}{2}}\sin(\theta)\frac{1}{\sqrt{dr}}J_l(dr)P_n^{m'}\cos(\theta)\sin(\theta),$$

$$M_{34}(r)=-\left(T_1r^2\right)^{-\frac{1}{2}}\left(\left(l+\frac{1}{2}\right)\frac{1}{\sqrt{dr}}J_l(dr)-\frac{1}{\sqrt{dr}}J_{l+1}(dr)\right)P_n^m\cos(\theta). \tag{10}$$

Making use of strain displacements relations and the Eq. (5) we obtain the fluid pressure is given by

$$s=\left(C_1M_{41}(r)+C_2M_{42}(r)+C_3M_{43}(r)+C_4M_{44}(r)\right)\cos(m\phi)e^{i\omega t}. \tag{11}$$

Where,

$$M_{41}(r)=\left(Q+R\delta_1^2\right)\left(T_1r^2\right)^{-\frac{1}{2}}\{\sin(\theta)\left[(-\cos(\theta)\sin(\theta))(1-\cos(\theta))^{-\frac{1}{2}}\frac{P_n^{m'}\cos(\theta)}{P_n^m\cos(\theta)}\sin(\theta)\right]+$$
$$\frac{\sin(\theta)}{P_n^m\cos(\theta)}\left(\left(P_n^{m''}\cos(\theta)\sin^2(\theta)-P^{m'}_{n}\cos(\theta)\cos(\theta)\right)\right)+m^2\frac{J_l(pr)}{\sqrt{pr}}+\left(l^2+\frac{1}{4}-pr^2\right)\frac{1}{\sqrt{pr}}J_l(pr)+$$
$$\left(l-\frac{3}{2}\right)\sqrt{pr}J_{l+1}(pr)\}P_n^m\cos(\theta),$$
$$M_{42}(r)=\left(Q+R\delta_1^2\right)\left(T_1r^2\right)^{-\frac{1}{2}}\{\sin(\theta)\left[(-\cos(\theta)\sin(\theta))(1-\cos(\theta))^{-\frac{1}{2}}\frac{q_n^{m'}\cos(\theta)}{q_n^m\cos(\theta)}\sin(\theta)\right]+$$
$$\frac{\sin(\theta)}{q_n^m\cos(\theta)}\left(\left(q_n^{m''}\cos(\theta)\sin^2(\theta)-q^{m'}_{n}\cos(\theta)\cos(\theta)\right)\right)+m^2\frac{J_l(qr)}{\sqrt{qr}}+\left(l^2+\frac{1}{4}-qr^2\right)\frac{1}{\sqrt{qr}}J_l(qr)+$$
$$\left(l-\frac{3}{2}\right)\sqrt{qr}J_{l+1}(qr)\}q_n^m\cos(\theta),$$
$$M_{43}(r)=0,\ M_{44}(r)=0. \tag{12}$$

3. Boundary condition and frequency equation

The sphere is under consideration rigidly fixed, we have the following boundary conditions on the surface $r=a$,

$$u_r=0,\quad u_\theta=0,\quad u_\phi=0,\quad s=0.\ \text{(for pervious surface)}$$
$$u_r=0,\quad u_\theta=0,\quad u_\phi=0,\quad \frac{\partial s}{\partial r}=0\ \text{(for impervious surface)} \tag{13}$$

Eq. (9) and (13) result in a system of four homogeneous equations in c_1, c_2, c_3 and c_4. For a nontrivial solution, determinant of coefficients matrix is zero. Accordingly, we obtain the following frequency equation for a pervious surface.

$$\begin{vmatrix} M_{11}(a) & M_{12}(a) & M_{13}(a) & M_{14}(a) \\ M_{21}(a) & M_{22}(a) & M_{23}(a) & 0 \\ M_{31}(a) & M_{32}(a) & M_{33}(a) & M_{34}(a) \\ M_{41}(a) & M_{42}(a) & 0 & 0 \end{vmatrix}=0. \tag{14}$$

4. Numerical results

The frequency equation is investigated by introducing the non-dimensional quantities given below:

$$a_1=\frac{P}{H},\ a_2=\frac{Q}{H},\ a_3=\frac{R}{H},\ a_4=\frac{N}{H}$$
$$d_1=\frac{\rho_{11}}{\rho},\ d_2=\frac{\rho_{12}}{\rho},\ d_3=\frac{\rho_{22}}{\rho},\ \tilde{z}=\left((\frac{V_0}{Vs})^2\right)$$
$$\rho=\rho_{11}+2\rho_{12}+\rho_{22};\quad H=P+2Q+R$$

$$V_0^2=\frac{H}{\rho}\,;\; m=\frac{c}{c_0}\,;\; c=\frac{\omega}{k}\,;\; c_0^2=\frac{N}{\rho} \tag{15}$$

Employing the non-dimensional quantities in the frequency equation (14), we obtain implicit relation between non-dimensional phase velocity (m_0) and non-dimensional wave number (ka), non-dimensional phase velocity is computed against wave number for three types of poroelastic solids. The values of θ and ϕ are taken to be $\frac{\pi}{6}$ arbitrarily. The value of m is taken to be 1 and the values of n is taken to be 2 following the paper [9]. For numerical work, three materials are considered. Spherical shell- I is made up of sandstone saturated with kerosene [10] while spherical shell- II is made up of sandstone saturated with water [11] and third solid is bone. The values of bone poroelastic parameters A,N,Q,R and the densities are computed following the paper [12].The values of Young's modulus and Poisson ratio are taken to be 3×10^6 and 0.28, respectively as suggested in the paper [12]. The numerical process is performed in MATLAB and the values are depicted in Fig.1. The values for pervious boundary and impervious boundary are equal. This means nature of boundary does not have an influence and the phase velocity. Fig.1 shows the plots of non-dimensional phase velocity against the non-dimensional wave number. From this figure, it is clear that material-1 values are much greater than that of material-2. Fluids present in these materials are causing this difference. The values of bone are greater than that of both material-1 and material-2. Moreover, as wave number increases non-dimensional phase decreases for all the poroelastic solids.

Table.1: Material parameters

Material parameters	a_1	a_2	a_3	a_4	d_1	d_2	d_3	$\tilde{x}$	$\tilde{y}$	$\tilde{z}$
Material-1	0.843	0.065	0.028	0.234	0.901	-0.001	0.101	0.999	4.763	3.851
Material-2	0.96	0.006	0.028	0.412	0.887	0	0.123	0.913	4.347	2.129
Bone	0.782	0.091	0.035	0.167	0.92	0	0.08	0.96	3.88	5.55

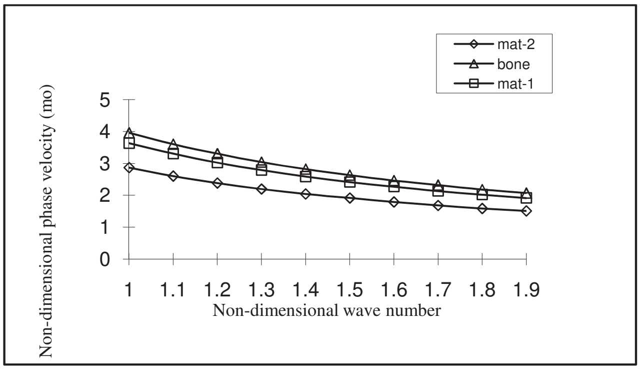

Fig.1. Non-dimensional wave number against non-dimensional phase velocity

5. Conclusions

The flexural vibrations in an isotropic poroelastic sphere subjected to rigidly fixed boundary conditions are investigated in the framework of Biot's theory in the case of pervious surface. Phase velocity against wave number is investigated for three types of poroelastic solids of the three solids considered here, two are sandstone spherical and third one is bone element. The values of material 1 are greater than that of material 2. Fluids present in these materials are causing this difference. Bone values are greater than that of both materials. It is also evident that nature of surface does not have any influence over the value.

References

[1] R. Kumar, Axially symmetric vibrations of a fluid-filled spherical Shell, Acustica 21 (1969) 143– 149.
[2] S. Paul, A note on the Radial vibrations of a sphere of poroelastic material, Indian J. Pure Appl. Math. 7 (1974) 469–475.
[3] S.A. Shah, M. Tajuddin, On axially symmetric vibrations of fluid filled poroelastic spherical shells, Open J. Acoust. 1 (2011) 15–26.
[4] M.A. Biot, The theory of propagation of elastic waves in fluid-saturated porous solid, J.Acous. Soc.Am. 28 (1956) 168–178.
[5] G. Rajitha, P. Malla Reddy, Investigation of flexural vibrations in poroelastic elliptic cone, published to Elsevier proceedings, International Conference on Mathematical Sciences, ICMS-2014.
[6] B. Shanker, C.N. Nath, S.A. Shah, P.M. Reddy, Vibrations in a fluid-loaded poroelastic hollow cylinder surrounded by a fluid in Plane- strain from, Int. J. of Appl. Mech Engg. 18(1) (2013) 189-216.
[7] D.C. Gazis, Exact analysis of plane-strain vibrations of thick-walled hollow Cylinders, J. Acoust. Soc. America. 30 (1958) 786-794.
[8] D.C. Gazis, Three dimensional investigation of the propagation of wave in hollow circular cylinders, I., Analytical Foundation, J. Acoust.Soc. Am., USA, 31 (1959) 568-573.
[9] R.H. Rand Torsional vibrations of elastic prolate spheroids, J. Acoust. Soc. Am. 44(3) (1968) 749- 751.
[10] C.H.Yew, P.N. Jogi, Study of wave motions in fluid-saturated porous rocks, J. Acoust. Soc Am. 60 (1976) 2–8.
[11] I. Fatt, The Biot-willis elastic coefficients for a sandstone, J. Appl. Mech. 26 (1959) 296–297.
[12] J.L. Nowinski, C.F. Davis, Propagation of longitudinal waves in circularly cylindrical bone elements, Transactions of ASME, Journal of Applied Mechanics 38(3) (1971) 578-584.